Chapter 7 : Pythagoras Theorem (Part 1)

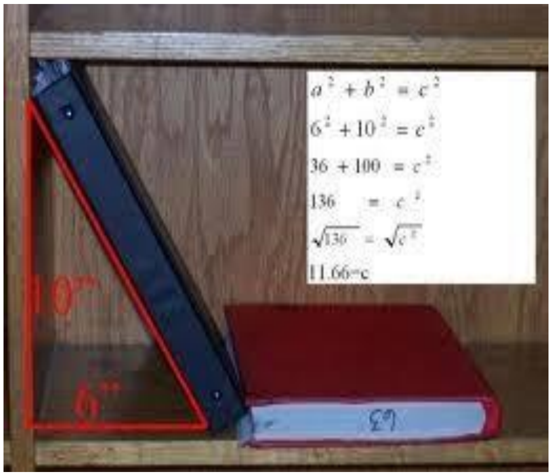
At the end of this chapter, you will learn about:

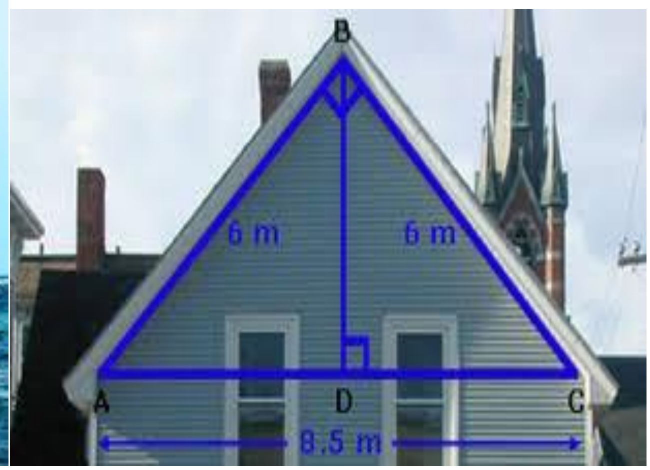
- The Pythagoras Theorem
- Its applications to right angled triangles

Here is the picture of Pythagoras. He was a Greek philosopher. He has provided a proof of a theorem which bears his name and he has also found methods to find the Pythagorean triples, which are sets of 3 whole numbers which make up the sides of right-angled triangles.

Real-Life applications of Pythagoras Theorem

Pythagoras' theorem helps to calculate the length of the diagonal connecting two given straight lines. This application of the theorem is generally used in architecture, carpentry and other construction works.



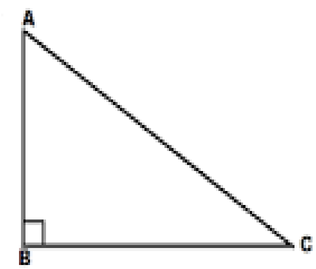


Learning Outcome:

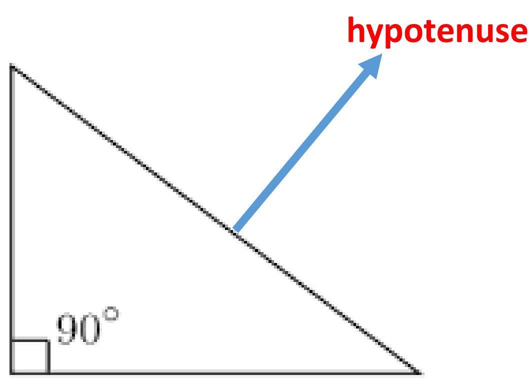
At the end of this lesson, you will be able to understand

- the Pythagoras Theorem and
- how to apply it to right-angled triangles.

Recall:



A right-angled triangle is a triangle containing a right angle, that is, a triangle having an angle of 90°.



The side opposite the right-angle (the angle of 90^0) is called the hypotenuse and it is in fact the longest side in the triangle.

Exercises for Practice- Exercise 7A

Identify the hypotenuse in the following right-angled triangles.

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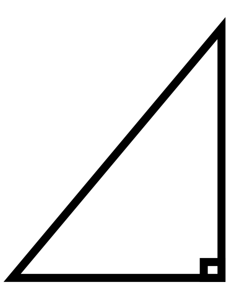

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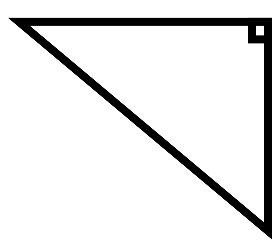

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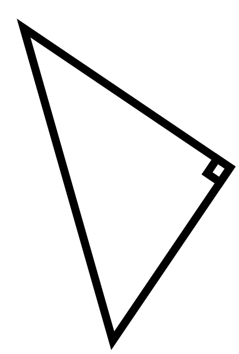

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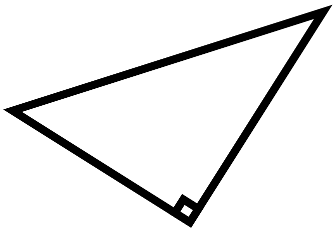

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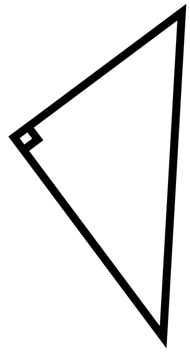

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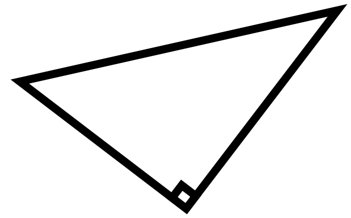

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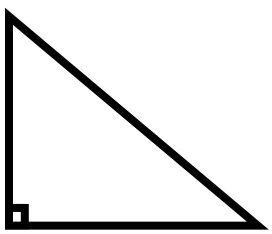

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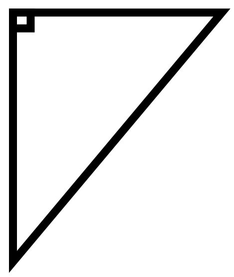

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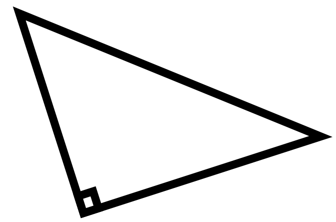
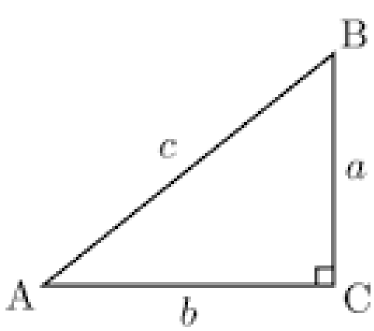


Pythagoras theorem



In any right-angled triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.

$$c^2 = a^2 + b^2$$

Pythagorean triples

If three integer values satisfy the Pythagoras theorem, we call the 3 numbers the Pythagorean triple.

Example 1:

Do 3, 4 and 5 form the Pythagorean triple?

Solution:

Consider the following 3 numbers: 3, 4 and 5.

The largest value is 5.

So, $5^2 = 25$

Now, $3^2 + 4^2 = 9 + 16 = 25$

Hence, $3^2 + 4^2 = 5^2$

3, 4 and 5 are called the Pythagorean triple.

Example 2:

Do 7, 8 and 6 form the Pythagorean triple?

Solution:

Consider the following 3 numbers: 7, 8 and 6.

The largest value is 8.

So, $8^2 = 64$

Now, $7^2 + 6^2 = 49 + 36 = 85$

Hence, $7^2 + 6^2 \neq 8^2$

6, 7 and 8 do not form the Pythagorean triple.

Exercises for Practice- Exercise 7B

Check whether the following sets of numbers form the Pythagorean triple.

(a) 12, 5, 13

(b) 24, 25, 26

(c) 6, 8, 10

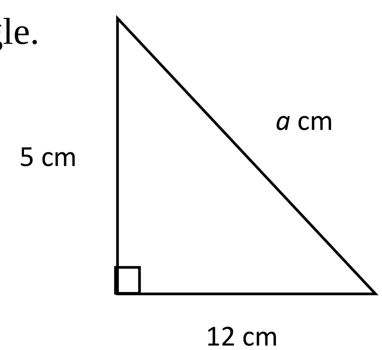
(d) 13, 15, 17

(e) 12, 35, 37

(f) 41, 9 and 40

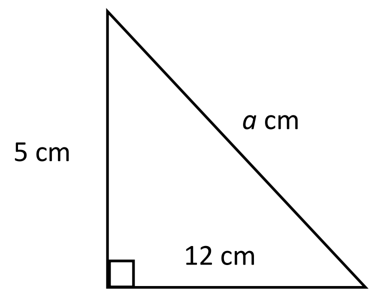
Example 3:

Find the value of *a* in the given triangle.



Solution:

Using Pythagoras Theorem,



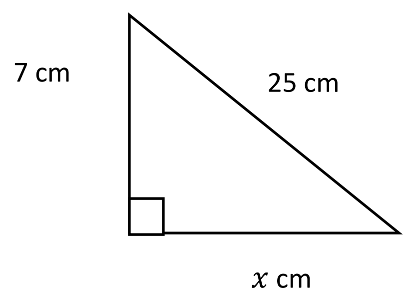
$a^2 = 12^2 + 5^2$

$a^2 = 144 + 25 = 169$

$a = \sqrt{169} = 13\ cm$

Example 4:

Find the length of the unknown side x.



Solution:

Using the Pythagoras' theorem,

We have $25^2 = 7^2 + x^2$

$$625 = 49 + x^2$$

$$x^2 = 625 - 49 = 576$$

$$x = \sqrt{576} = 24\ cm$$

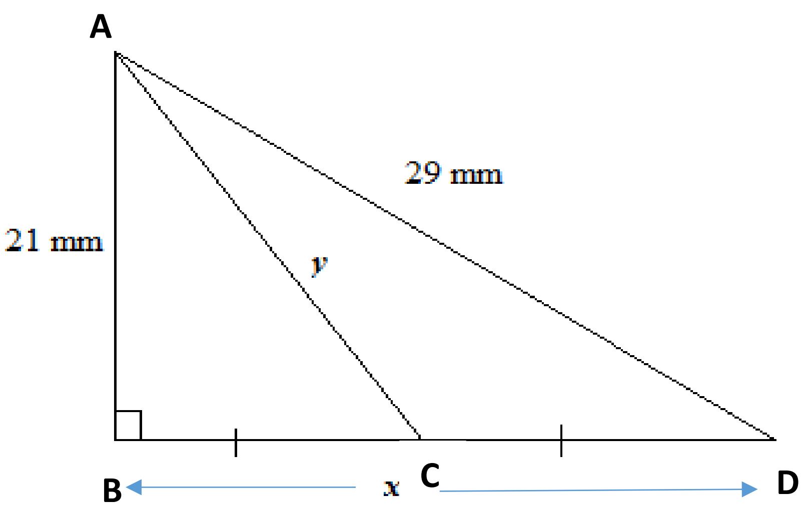
 Have you been able to observe when we need to ‘add’ or when we need to ‘subtract’ the square of the sides?

Hint:

When we need to find the longest side of the right-angled triangle, we will have to ‘add’.

If the longest side is given and we need to find one of the shorter sides, then we will have to ‘subtract’.

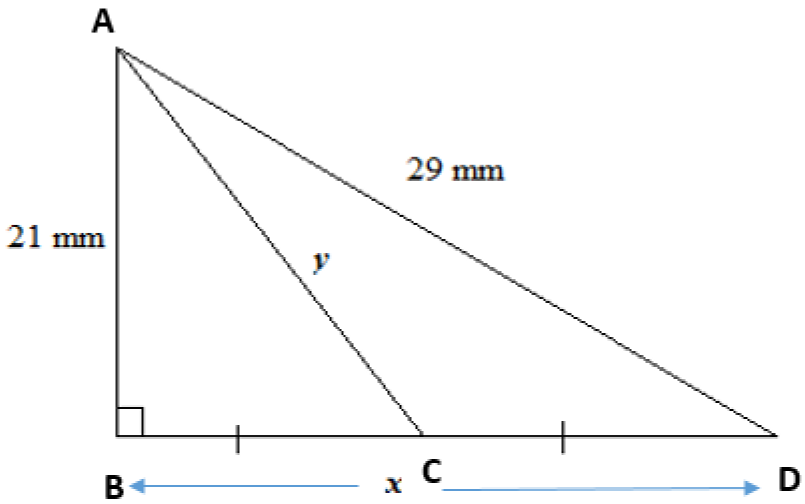
Example 5:



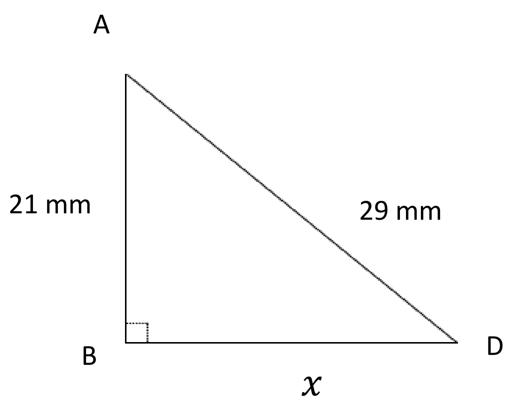
Find the value of x and y in the given figure.

Solution:

We need to consider the triangles separately to find the values of x and y.



We first consider triangle ABD.



$$29^2 = 21^2 + x^2$$

$$841 = 441 + x^2$$

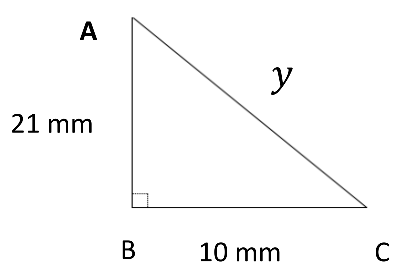
x^2 = 841 – 441 = 400

$$x = \sqrt{400} = 20\ mm$$

Now, BC = CD

BC = (½ $\times 20)\ mm = 10\ mm$

We now consider triangle ABC.



$$y^2 = 21^2 + 10^2$$

$$y^2 = 441 + 100$$

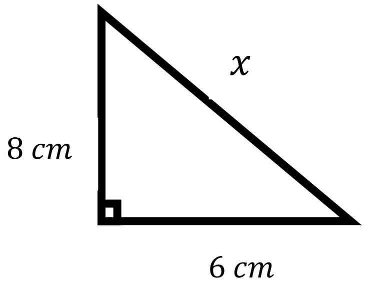
$$y^2 = 541$$

$$y = \sqrt{541}\ mm$$

Exercises for Practice- Exercise 7C

1. Find the value of the unknown side in the given triangles. Leave your answer in terms of $\sqrt{\ }$ if it is not exact.

(a)



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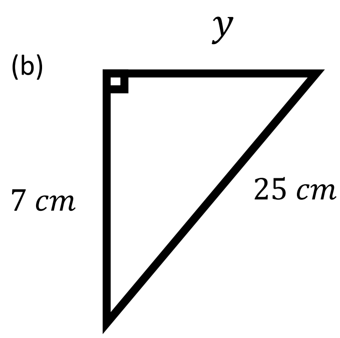

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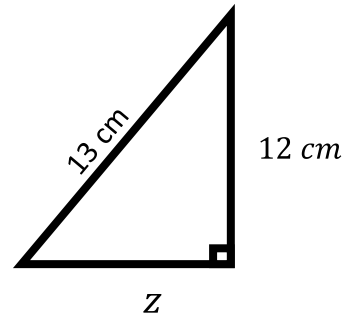

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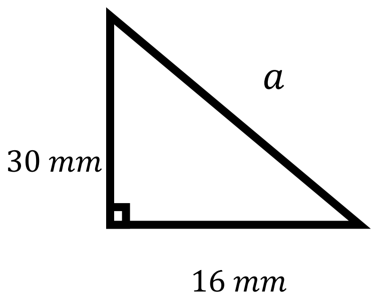

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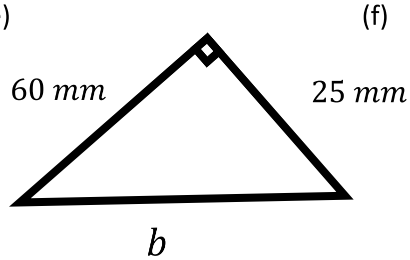

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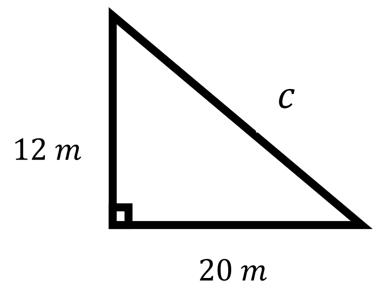
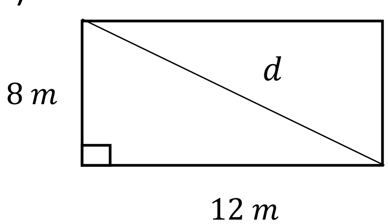


2. Find the value of the unknown side(s) in the given diagrams. Leave your answer(s) in terms of $\sqrt{\ }$ if it is not exact.

(a)



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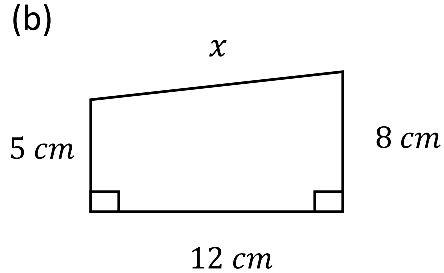

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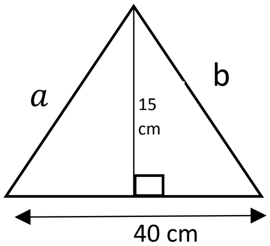

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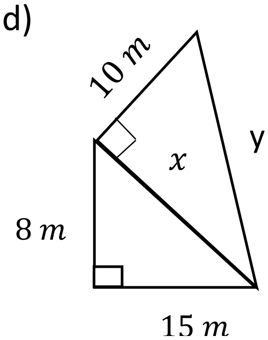

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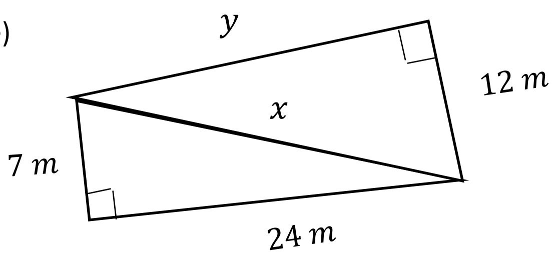

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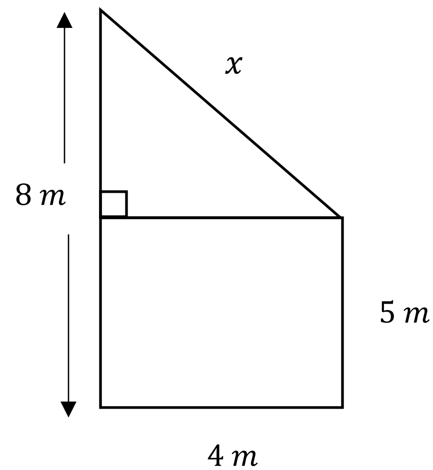

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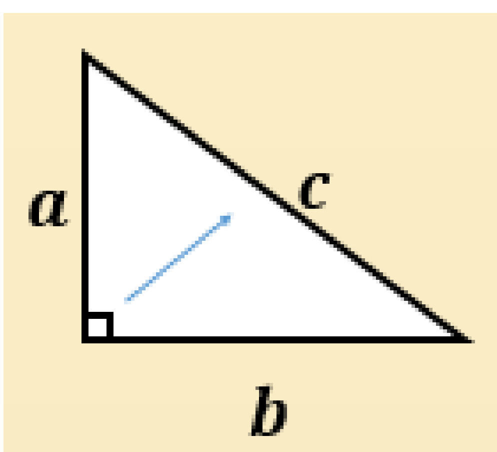


$$c^2 = a^2 + b^2$$

In any right-angled triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.

Links for practice:

https://www.ozarktigers.org/cms/lib/MO01910080/Centricity/Domain/559/pythagorean_theorem_worksheet.pdf

http://templatelab.com/pythagorean-theorem-worksheet/

http://www.math-aids.com/Pythagorean_Theorem/

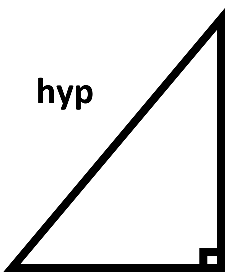
http://pythagoras.nu/pythagorean-triples/

Answers to Exercises:

Exercise 7A

1.

(a)



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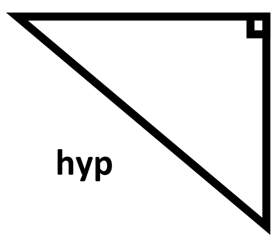

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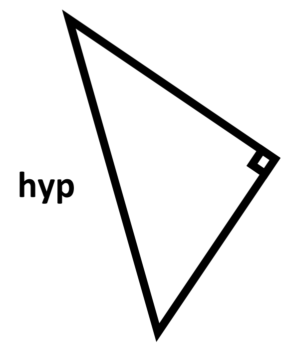

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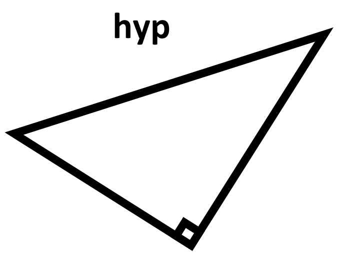

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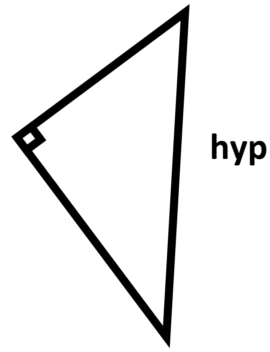

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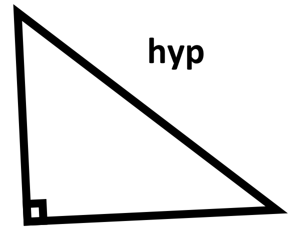

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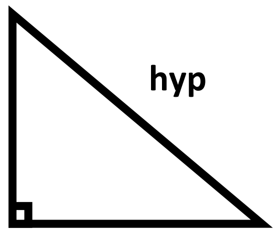

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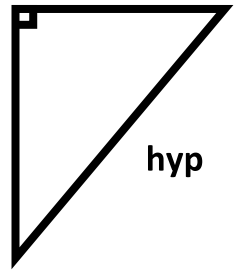

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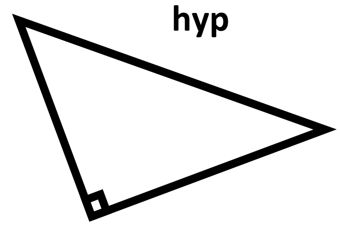

Exercise 7B

(a) Yes 12, 5 and 13 form the Pythagorean triple.

(b) No 24, 25 and 26 do not form the Pythagorean triple.

(c) Yes 6, 8 and 10 form the Pythagorean triple.

(d) No 13, 15 and 17 do not form the Pythagorean triple.

(e) Yes 12, 35 and 37 form the Pythagorean triple.

(f) Yes 41, 9 and 40 form the Pythagorean triple.

Exercise 7C

1. (a) $x = 10\ cm$ (b) $y = 24\ cm$ (c) $z = 5\ cm$

   (d) $a = 34\ mm$ (e) $b = 65\ mm$ (f) $c = \sqrt{544}\ m$

2. (a) $d = \sqrt{208}\ m$ (b) $x = \sqrt{153}\ cm$

   (c) $a = b = 25\ cm$ (d) $x = 17\ m, y = \sqrt{389}\ m$

   (e) $x = 25\ m,\ y = \sqrt{481}\ m$ (f) $x = 5\ m$