Indices part 1

In this chapter, you are going to

- Apply the two rules: $(ab)^n = a^n b^n$ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
- Learn about negative indices
- Learn about fractional indices
- Use fractional indices to evaluate square roots and cube roots

Recap

1. Numbers written in index form

$$81 = 3 \times 3 \times 3 \times 3 = 3^4$$

$3 \times 3 \times 3 \times 3$ → Expanded form

3^4 → Index form

3^4 is read as 3 to the power or index of 4 where **3** is called the base.

2. Multiplication law

$$\boldsymbol{a^m \times a^n = a^{m+n}}$$

Examples:

(a) $a^3 \times a^2 = a^{3+2} = a^5$

(b) $4^5 \times 4^2 = 4^{5+2} = 4^7$

(c) $3a^3 \times 2a^2 = (3 \times 2)a^{3+2} = 6a^5$

3. Division law

$$\boldsymbol{a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}, a \neq 0}$$

Examples

(a) $\boldsymbol{a^5 \div a^2 = a^{5-2} = a^3}$

(b) $\boldsymbol{\frac{3^7}{3^2} = 3^{7-2} = 3^5}$

(c) $\boldsymbol{\frac{12m^5}{3m^4} = (\frac{12}{3})m^{5-4} = 4m^1}$ **or simply** $\boldsymbol{4m}$

4. Power law

$$(a^m)^n = a^{mn}$$

Examples

(a) $(m^2)^4 = m^{2\times 4} = m^8$

(b) $(5^3)^2 = 5^{3\times 2} = 5^6$

(c) $(m^2)^4 \times (m^3)^5 = m^8 \times m^{15} = m^{23}$

(d) $\frac{(p^3)^7 \times (p^4)^3}{(p^2)^4} = \frac{p^{21} \times p^{12}}{p^8} = \frac{p^{33}}{p^8} = p^{25}$

5. Meaning of a^0

Any number raised to the power of zero is 1

$$\boldsymbol{a^0 = 1}$$

Examples

(a) $m^0 = 1$

(b) $3^0 = 1$

(c) $3m^0 = 3$

(d) $b^5 \div b^5 = b^0 = 1$

So far we have done a recap of the various laws of indices you have learned in grade 7. Now we are going to learn some more laws and rules of indices in grade 8.

1. Consider the following

$(\boldsymbol{ab})^3 = (\boldsymbol{ab}) \times (\boldsymbol{ab}) \times (\boldsymbol{ab})$

$= \boldsymbol{a} \times \boldsymbol{b} \times \boldsymbol{a} \times \boldsymbol{b} \times \boldsymbol{a} \times \boldsymbol{b}$

$= \boldsymbol{a}^3\boldsymbol{b}^3$

So we conclude that

$$(\boldsymbol{ab})^3 = \boldsymbol{a}^3\boldsymbol{b}^3$$

Similarly

$\left(\frac{\boldsymbol{a}}{\boldsymbol{b}}\right)^2 = \left(\frac{\boldsymbol{a}}{\boldsymbol{b}}\right) \times \left(\frac{\boldsymbol{a}}{\boldsymbol{b}}\right) = \frac{\boldsymbol{a} \times \boldsymbol{a}}{\boldsymbol{b} \times \boldsymbol{b}} = \frac{\boldsymbol{a}^2}{\boldsymbol{b}^2}$

We conclude that

$$\left(\frac{\boldsymbol{a}}{\boldsymbol{b}}\right)^2 = \frac{\boldsymbol{a}^2}{\boldsymbol{b}^2}$$

Rule 1 :

$$(\boldsymbol{ab})^{\boldsymbol{n}} = \boldsymbol{a}^{\boldsymbol{n}}\boldsymbol{b}^{\boldsymbol{n}}$$

Rule 2:

$$\left(\frac{\boldsymbol{a}}{\boldsymbol{b}}\right)^{\boldsymbol{n}} = \frac{\boldsymbol{a}^{\boldsymbol{n}}}{\boldsymbol{b}^{\boldsymbol{n}}}$$

Examples

Simplify the following:

(a) $(2m)^3$

(b) $(3p)^2$

(c) $(2m)^0$

(d) $2(mn)^0$

(e) $\left(\frac{2}{p}\right)^2$

(f) $\left(\frac{2m}{n}\right)^2$

Solutions:

(a) $(2m)^3 = 2^3m^3 = 8m^3$ (according to rule 1, the power 3 belongs to both 2 and m)
(b) $(3p)^2 = 3^2p^2 = 9p^2$ (according to rule 1, the power 2 belongs to both 3 and p)
(c) $(2m)^0 = 2^0m^0 = 1 \times 1 = 1$ since $(2m)^0 = 1$
(d) $2(mn)^0 = 2 \times m^0 \times n^0 = 2 \times 1 \times 1 = 2$ since $2(mn)^0 = 2$
(e) $\left(\frac{2}{p}\right)^2 = \frac{2^2}{p^2} = \frac{4}{p^2}$ (according to rule 2, the power 2 belongs to both 2 and p)
(f) $\left(\frac{2m}{n}\right)^2 = \frac{2^2m^2}{n^2} = \frac{4m^2}{n^2}$ (according to rule 2, the power 2 belongs to 2 , m and n)

2. Negative indices

$$a^{-x} = \frac{1}{a^x}$$

Examples

Evaluate the following:

(a) 5^{-2}
(b) 2^{-3}
(c) $4y^{-3}$
(d) $3m^{-2}$
(e) $a^{-3} \times a^{-2}$
(f) $(2^3)^{-2}$

Solutions

(a) $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$
(b) $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$
(c) $4y^{-3} = 4\left(\frac{1}{y^3}\right) = \frac{4}{y^3}$
(d) $3m^{-2} = 3\left(\frac{1}{m^2}\right) = \frac{3}{m^2}$
(e) $a^{-3} \times a^{-2} = a^{-3+-2} = a^{-5} = \frac{1}{a^5}$
(d) $\frac{3^4}{3^7} = 3^{4-7} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$
(f) $(2^3)^{-2} = 2^{-6} = \frac{1}{2^6} = \frac{1}{64}$

3. Power law involving fractional indices.

$$(\boldsymbol{a^x})^{\frac{1}{y}} = \boldsymbol{a}^{x\times\frac{1}{y}}$$

Examples

Evaluate the following:

(a) $(2^3)^{\frac{1}{3}}$

(b) $(5^4)^{\frac{1}{2}}$

(c) $27^{\frac{1}{3}}$

(d) $125^{\frac{2}{3}}$

(e) $8000^{\frac{2}{3}}$

Solutions

(a) $(2^3)^{\frac{1}{3}} = 2^{3\times\frac{1}{3}} = 2^1 = 2$

(b) $(5^4)^{\frac{1}{2}} = 5^{4\times\frac{1}{2}} = 5^2 = 25$

(c) $27^{\frac{1}{3}} = 3^{3\times\frac{1}{3}} = 3^1 = 3$

(d) $125^{\frac{2}{3}} = 5^{3\times\frac{2}{3}} = 5^2 = 25$

(e) $8000^{\frac{2}{3}} = (2^6 \times 5^3)^{\frac{2}{3}} = (2^6)^{\frac{2}{3}} \times (5^3)^{\frac{2}{3}} = 2^{3\times 2} \times 5^2 = 16 \times 25 = 400$

Exercises for practice

1. Simplify the following.

(a) $y^4 \times y^2$ (b) $5y^3 \times 2y^2$

(c) $xy^3 \times x^2y$ (d) $5a^3b^5 \times 2ab^2$

(e) $y^3 \times x \times y^2 \times x^4$ (f) $3m^3 \times 2n \times 4m^2n^2 \times m^4$

2. Simplify the following.

(a) $y^4 \div y^2$ (b) $p^7 \div p^2$

(c) $\frac{m^4}{m^2}$ (d) $\frac{a^5b^6}{ab^3}$

(e) $\frac{x^8 \times x}{x^2 \times x^4}$ (f) $\frac{5m^3 \times 4m^3}{2m^2 \times m^4}$

3. Simplify the following, giving your answer as positive index.

(a) m^{-2} (b) $2p^{-3}$

(c) $y^{-3} \times y^2$ (d) $3m^{-2} \times m^{-3}$

(e) $a^{-3} \div a^{-2}$ (f) $(2a^3)^{-2}$

4. Simplify the following.

(a) $(3m)^4$ (b) $(pq^4)^2$

(c) $(a^2b^5)^3$ (d) $(3m^3)^4 \times (2m^2)^2$

(e) $5(p^5q)^3 \div (p^3q)^3$ (f) $(p^3q)^7 \times (p^5)^3 \times q^5$

(g) $\left(\frac{m}{n}\right)^5$ (h) $\left(\frac{p^4}{q^2}\right)^5$

(i) $15(ab^3)^7 \times 3\left(\frac{a}{b}\right)^5$ (j) $12(mn)^7 \times 3mn^5 \div (3m)^4$

5. Evaluate the following

(a) $(4^3)^{\frac{1}{3}}$ (b) $(3^4)^{\frac{1}{2}}$

(c) $(64a^3)^{\frac{1}{3}}$ (d) $(125p^3)^{\frac{2}{3}}$

(e) $\left(\frac{16}{81}\right)^{\frac{1}{2}}$ (f) $\left(\frac{25}{16}\right)^{-\frac{1}{2}}$

Solutions

1. (a) y^6 (b) $10y^5$ (c) x^3y^4 (d) $10a^4b^7$ (e) x^5y^5 (f) $24\,m^9n^3$
2. (a) y^2 (b) p^5 (c) m^2 (d) a^4b^3 (e) x^3 (f) 10
3. (a) $\frac{1}{m^2}$ (b) $\frac{2}{p^3}$ (c) $\frac{1}{y}$ (d) $\frac{3}{m^5}$ (e) $\frac{1}{a}$ (f) $\frac{1}{4a^6}$
4. (a) $81m^4$ (b) p^2q^8 (c) a^6b^{15} (d) $324m^{16}$ (e) $5p^6$ (f) $p^{36}q^{12}$

   (g) $\frac{m^5}{n^5}$ (h) $\frac{p^{20}}{q^{10}}$ (i) $45a^{12}b^{16}$ (j) $\frac{4}{9}m^4n^{12}$
5. (a) 4 (b) 9 (c) $4a$ (d) $25p^2$ (e) $\frac{4}{9}$ (f) $\frac{4}{5}$

Links for reference

1. **http://mathematics.laerd.com/maths/indices-intro.php**
2. **https://revisionmaths.com/advanced-level-maths-revision/pure-maths/algebra/indices**
3. **https://www.youtube.com/watch?v = ZFUuoTDeZhk**

Indices part (II)

Square roots and cube roots

Fractional indices

$$\sqrt{a} = a^{\frac{1}{2}}$$

$$\sqrt[3]{a} = a^{\frac{1}{3}}$$

Note

$\sqrt{a}$ can be written as $a^{\frac{1}{2}}$ and $\sqrt[3]{a}$ can be written as $a^{\frac{1}{3}}$

Examples

Evaluate the following

(a) $\sqrt{81}$ (b) $\sqrt{144}$ (c) $\sqrt[3]{64}$ (d) $\sqrt[3]{27000}$

Solutions

(a) $\sqrt{81} = (81)^{\frac{1}{2}}$ (we first write 81 to the power $\frac{1}{2}$)

$= 3^{4\times\frac{1}{2}}$ (81 is then written as product of its prime factors in index form)

$= 3^2$ (use power law for evaluation)

$= 9$

(b) $\sqrt{144} = (144)^{\frac{1}{2}}$ (we first write 144 to the power $\frac{1}{2}$)

$= (2^4 \times 3^2)^{\frac{1}{2}}$ (81 is then written as product of its prime factors in index form)

$= 2^{4\times\frac{1}{2}} \times 3^{2\times\frac{1}{2}}$ (use power law for evaluation)

$= 2^2 \times 3$

$= 12$

(c) $\sqrt[3]{64} = (64)^{\frac{1}{3}}$ (we first write 64 to the power $\frac{1}{3}$)

$= 2^{6\times\frac{1}{3}}$ (64 is then written as product of its prime factors in index form)

$= 2^2$ (use power law for evaluation)

$= 4$

(d) $\sqrt[3]{27000} = (27000)^{\frac{1}{3}}$ (we first write 27000 to the power $\frac{1}{3}$)

$= (3^3 \times 10^3)^{\frac{1}{3}}$ (64 is then written as product of its prime factors in index form)

$= 3^{3\times\frac{1}{3}} \times 10^{3\times\frac{1}{3}}$ (use power law for evaluation)

$= 3 \times 10$

$= 30$

Standard values

$$\sqrt{100} = 10$$

$$\sqrt{10\,000} = 100$$

$$\sqrt{1\,000\,000} = 1\,000$$

These standard values are used to calculate square roots of large numbers or very small numbers.

Example

1. Evaluate the following.
   (a) $\sqrt{400}$
   (b) $\sqrt{250000}$
   (c) $\sqrt{0.81}$
   (d) $\sqrt{0.0000144}$

Solution

(a) $\sqrt{400} = \sqrt{4 \times 100}$ (since $\sqrt{100} = 10$)

$= \sqrt{4} \times \sqrt{100}$

$= 2 \times 10$

$= 20$

(b) $\sqrt{250000} = \sqrt{25 \times 10\,000}$ (since $\sqrt{10\,000} = 100$)

$= \sqrt{25} \times \sqrt{10\,000}$

$= 5 \times 100$

$= 500$

(c) $\sqrt{0.81} = \sqrt{\frac{81}{100}}$ (since $\sqrt{100} = 10$)

$= \dfrac{9}{10}$

$= 0.9$

(d) $\sqrt{0.000\,0144} = \sqrt{\frac{144}{1\,000\,000}}$ (since $\sqrt{1\,000\,000} = 1000$)

$= \dfrac{12}{1000}$

$= 0.012$

2. Evaluate the following, given that $\sqrt{3} = 1.73$ and $\sqrt{30} = 5.48$

(a) $\sqrt{3\,000}$

(b) $\sqrt{0.03}$

(c) $\sqrt{0.3}$

Solutions

(a) $\sqrt{3\,000} = \sqrt{3 \times 1000}$ (we cannot evaluate $\sqrt{1000}$, we use $\sqrt{100}$)

$= \sqrt{30 \times 100}$

$= 5.48 \times 10$

$= 54.8$

(b) $\sqrt{0.03} = \sqrt{\frac{3}{100}}$

$= \sqrt{\dfrac{3}{100}}$

$= \dfrac{1.73}{10}$

$= 0.173$

(c) $\sqrt{0.3} = \sqrt{\frac{3}{10}}$ (we cannot evaluate $\sqrt{10}$, so we use $\sqrt{100}$)

$$= \sqrt{\frac{30}{100}}$$

$$= \frac{5.48}{10}$$

$$= 0.548$$

Square roots of fractions written as mixed number

Whenever we have to evaluate square roots of fractions written as mixed numbers, we first write the fraction as improper fraction, then we evaluate the square root

Examples

Evaluate the following

(a) $\sqrt{2\frac{7}{9}}$

(b) $\sqrt{1\frac{11}{25}}$

Solutions

(a) $\sqrt{2\frac{7}{9}} = \sqrt{\frac{25}{9}}$ the fraction is written as improper fraction

$$= \frac{5}{3}$$

$$= 1\frac{2}{3}$$

(b) $\sqrt{1\frac{11}{25}} = \sqrt{\frac{36}{25}}$ the fraction is written as improper fraction

$$= \frac{6}{5}$$

$$= 1\frac{1}{5}$$

Exercises for practice

1. Evaluate the following square root using indices

(a) $\sqrt{196}$ (b) $\sqrt{225}$

(c) $\sqrt{576}$ (d) $\sqrt[3]{125}$

(e) $\sqrt[3]{512}$ (f) $\sqrt[3]{8000}$

2. Evaluate the following

(a) $\sqrt{1600}$ (b) $\sqrt{360\,000}$

(c) $\sqrt{9\,000\,000}$ (d) $\sqrt{0.04}$

(e) $\sqrt{0.64}$ (f) $\sqrt{0.0081}$

3. Evaluate the following, given that $\sqrt{5} = 2.24$ and $\sqrt{50} = 7.07$

(a) $\sqrt{500}$ (b) $\sqrt{5000}$

(c) $\sqrt{50\,000}$ (d) $\sqrt{0.5}$

(e) $\sqrt{0.05}$ (f) $\sqrt{0.005}$

4. Evaluate the following, given that $\sqrt{7} = 2.65$ and $\sqrt{70} = 8.37$

(c) $\sqrt{700}$ (d) $\sqrt{7000}$

(c) $\sqrt{700\,000}$ (d) $\sqrt{0.007}$

(e) $\sqrt{0.0007}$ (f) $\sqrt{0.00007}$

5. Evaluate the following

(a) $\sqrt{\frac{9}{49}}$ (b) $\sqrt{2\frac{1}{4}}$

(c) $\sqrt{1\frac{13}{36}}$ (d) $\sqrt{5\frac{4}{9}}$

(e) $\sqrt{2\frac{2}{49}}$ (f) $\sqrt{2\frac{46}{49}}$

Solutions

1. (a) 14 (b) 15 (c) 24 (d) 5 (e) 8 (f) 20
2. (a) 40 (b) 600 (c) 3 000 (d) 0.2 (e) 0.8 (f) 0.09
3. (a) 22.4 (b) 70.7 (c) 224 (d) 0.707 (e) 0.224 (f) 0. 0707
4. (a) 26.5 (b) 83.7 (c) 837 (d) 0.0837 (e) 0.0265 (f) 0.00837
5. (a) $\frac{3}{7}$ (b) $1\frac{1}{2}$ (c) $1\frac{1}{6}$ (d) $2\frac{1}{3}$ (e) $1\frac{3}{7}$ (f) $1\frac{5}{7}$

Links

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